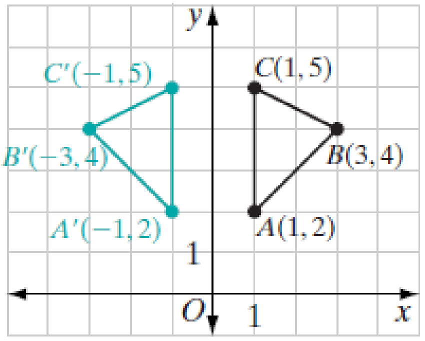
Reflection in the y-axis

In the figure, $\triangle ABC$ is reflected in the y-axis. Its image under the reflection is $\triangle A'B'C'$. From the figure, we see that:



$$A(1,2) \rightarrow A'(-1,2)$$
$$B(3,4) \rightarrow B'(-3,4)$$
$$C(1,5) \rightarrow C'(-1,5)$$

For each point and its image under a reflection in the y-axis, the y-coordinate of the image is the same as the y-coordinate of the point; the x-coordinate of the image is the opposite of the x-coordinate of the point. Note that for a reflection in the y-axis, the image of $(1, 2)$ is $(-1, 2)$ and the image of $(-1, 2)$ is $(1, 2)$.

A reflection in the y-axis can be designated as $r_{y\text{-axis}}$. For example, if the image of $(1, 2)$ is $(-1, 2)$ under a reflection in the y-axis, we can write:

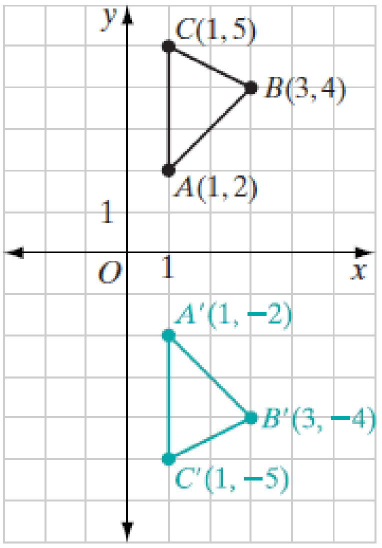
$$r_{y\text{-axis}}(1,2) = (-1,2)$$

From these examples, we form a general rule that can be proven as a theorem.

> Under a reflection in the y-axis, the image of $P(a, b)$ is $P'(-a, b)$.

Reflection in the x-axis

In the figure, $\triangle ABC$ is reflected in the x-axis. Its image under the reflection is $\triangle A'B'C'$. From the figure, we see that:



$$A(1,2) \rightarrow A'(1,-2)$$
$$B(3,4) \rightarrow B'(3,-4)$$
$$C(1,5) \rightarrow C'(1,-5)$$

For each point and its image under a reflection in the x-axis, the x-coordinate of the image is the same as the x-coordinate of the point; the y-coordinate of the image is the opposite of the y-coordinate of the point. Note that for a reflection in the x-axis, the image of $(1, 2)$ is $(1, -2)$ and the image of $(1, -2)$ is $(1, 2)$.

A reflection in the x-axis can be designated as $r_{x\text{-axis}}$. For example, if the image of $(1, 2)$ is $(1, -2)$ under a reflection in the x-axis, we can write:

$$r_{x\text{-axis}}(1,2) = (1,-2)$$

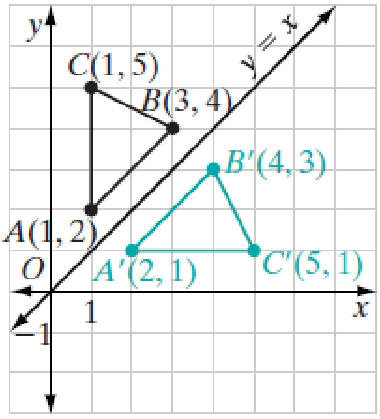
From these examples, we form a general rule that can be proven as a theorem.

> Under a reflection in the x-axis, the image of $P(a, b)$ is $P'(a, -b)$.

Reflection in the Line $y = x$

In the figure, $\triangle ABC$ is reflected in the line $y = x$. Its image under the reflection is $\triangle A'B'C'$. From the figure, we see that:

$$A(1, 2) \rightarrow A'(2, 1)$$
$$B(3, 4) \rightarrow B'(4, 3)$$
$$C(1, 5) \rightarrow C'(5, 1)$$



For each point and its image under a reflection in the line $y = x$, the x-coordinate of the image is the y-coordinate of the point; the y-coordinate of the image is the x-coordinate of the point. Note that for a reflection in the line $y = x$, the image of $(1, 2)$ is $(2, 1)$ and the image of $(2, 1)$ is $(1, 2)$.

A reflection in the line $y = x$ can be designated as $r_{y=x}$. For example, if the image of $(1, 2)$ is $(2, 1)$ under a reflection in $y = x$, we can write:

$$r_{y=x}(1, 2) = (2, 1)$$

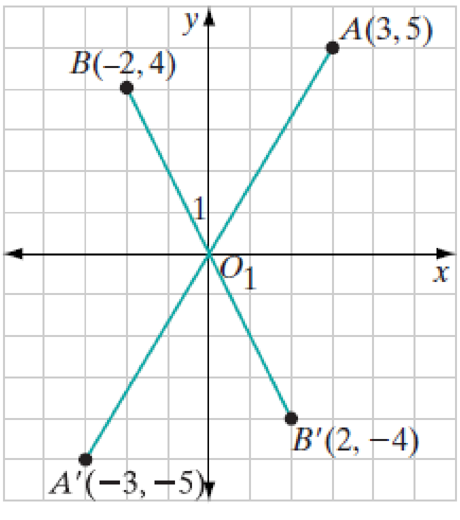
From these examples, we form a general rule that can be proven as a theorem.

> Under a reflection in the line $y = x$, the image of $P(a, b)$ is $P'(b, a)$.

Point Reflection in the Coordinate Plane

In the coordinate plane, the origin is the most common point that is used to used to define a point reflection.

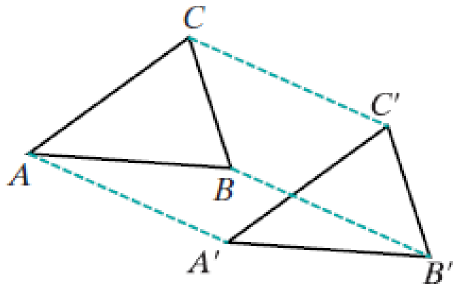
In the diagram, points $A(3, 5)$ and $B(-2, 4)$ are reflected in the origin. The coordinates of A', the image of A, are $(-3, -5)$ and the coordinates of B', the image of B, are $(2, -4)$. These examples suggest the following theorem.



> Under a reflection in the origin, the image of $P(a, b)$ is $P'(-a, -b)$.

DEFINITION

A **translation** is a transformation of the plane that moves every point in the plane the same distance in the same direction.



If $\triangle A'B'C'$ is the image of $\triangle ABC$ under a translation, $AA' = BB' = CC'$. It appears that the size and shape of the figure are unchanged, so that $\triangle ABC \cong \triangle A'B'C'$. Thus, under a translation, as with a reflection, a figure is congruent to its image. The following example in the coordinate plane shows that this is true. In the coordinate plane, the distance is given in terms of horizontal distance (change in the x-coordinates) and vertical distance (change in the y-coordinates).

DEFINITION

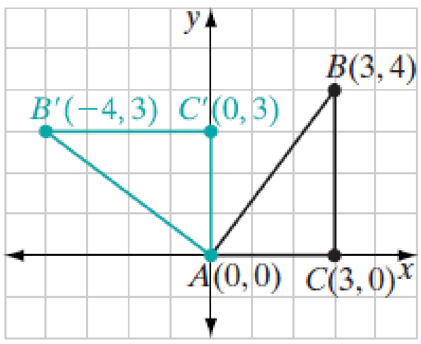
A **translation of a units in the horizontal direction and b units in the vertical direction** is a transformation of the plane such that the image of $P(x, y)$ is

$$P'(x + a, y + b).$$

Rotations in the Coordinate Plane

The most common rotation in the coordinate plane is a **quarter turn** about the origin, that is, a counterclockwise rotation of 90° about the origin.

In the diagram, the vertices of right triangle ABC are $A(0,0)$, $B(3,4)$ and $C(3,0)$. When rotated 90° about the origin, A remains fixed because it is the center of rotation. The image of C, which is on the x-axis and 3 units from the origin, is $C'(0, 3)$ on the y-axis and 3 units from the origin. Since $\overline{CB}$ is a vertical line 4 units long, its image is a horizontal line 4 units long and to the left of the y-axis. Therefore, the image of B is $B'(-4, 3)$. Notice that the x-coordinate of B' is the negative of the y-coordinate of B and the y-coordinate of B' is the x-coordinate of B.



The point $B(3, 4)$ and its image $B'(-4, 3)$ in the above example suggest a rule for the coordinates of the image of any point $P(x, y)$ under a counterclockwise rotation of 90° about the origin.

Under a counterclockwise rotation of 90° about the origin, the image of $P(a, b)$ is $P'(-b, a)$.