

Problem Solving

Level D

Pages 48 - 77

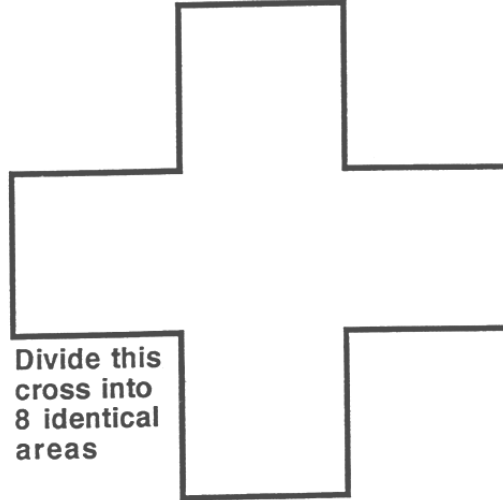
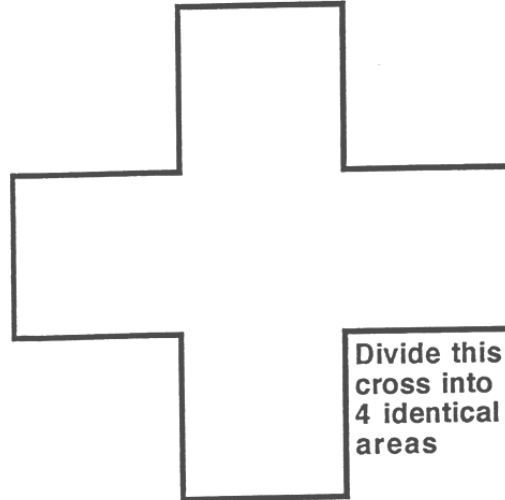
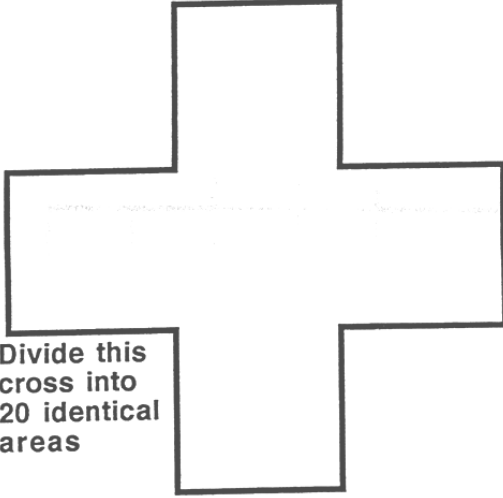
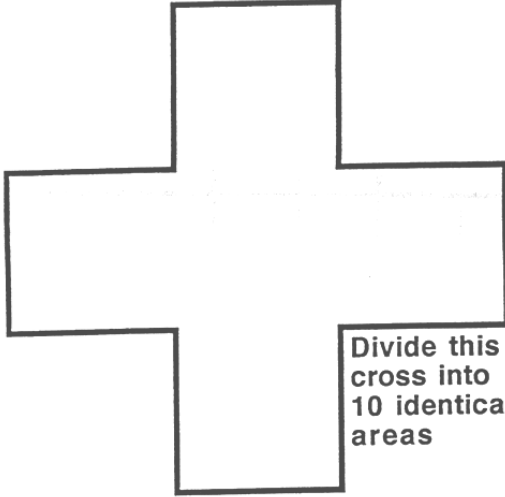
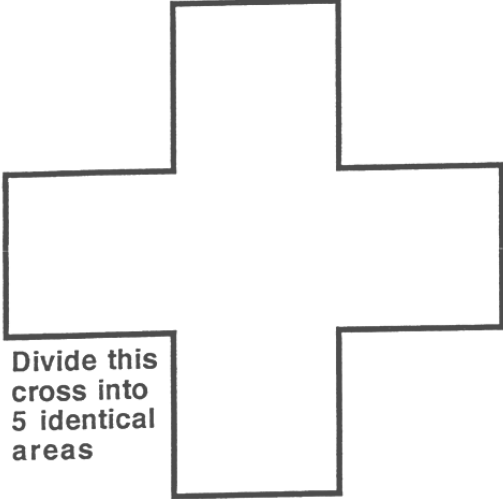
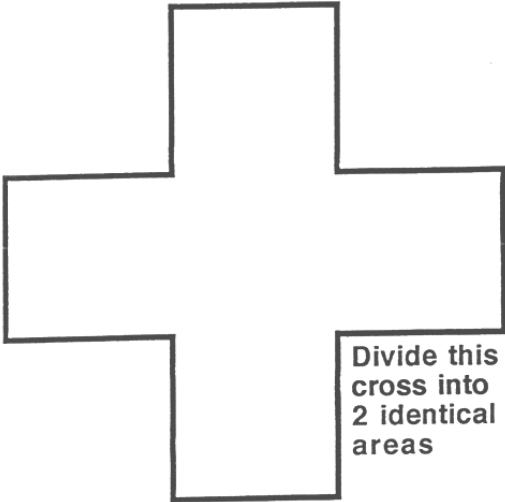
Strategies

Pages 48 – 52 - ‘Draw a picture, make a model’ (consolidate strategy)	Unit D3
Pages 53 – 55 - ‘Look for a pattern’ (consolidate strategy)	Unit D4
Pages 56 – 59 - ‘Guess, check and improve’ (consolidate strategy)	Unit D5
Pages 60 – 68 - ‘Be systematic’ (consolidate strategy)	Unit D6
Pages 69 – 71 - ‘Work backwards’ (new strategy)	Unit D7
Pages 72 – 77 - ‘Take a simpler case, make a table, spot a pattern’ (new strategy)	Unit D8

‘Unit’ refers to the Programme of Study

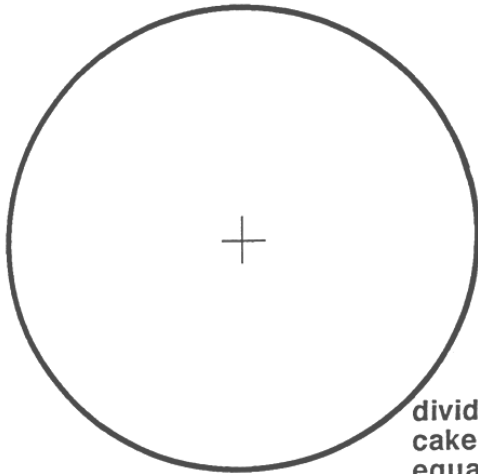
Divide them up equally

Problem Solving
draw a picture, make a model
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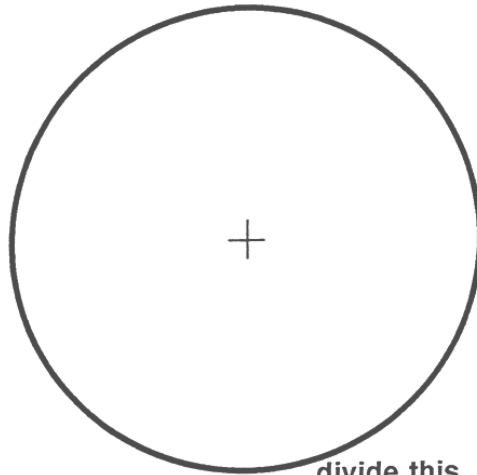


Equal shares of cake

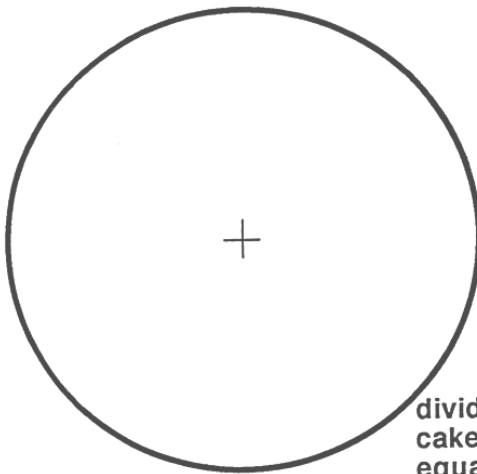
Problem Solving
draw a picture, make a model
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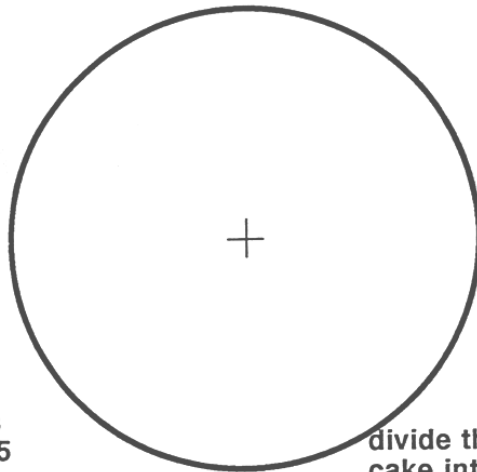
divide this
cake into 3
equal pieces



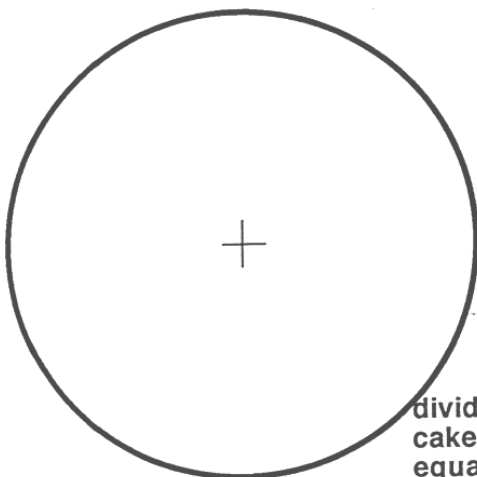
divide this
cake into 6
equal pieces



divide this
cake into 5
equal pieces

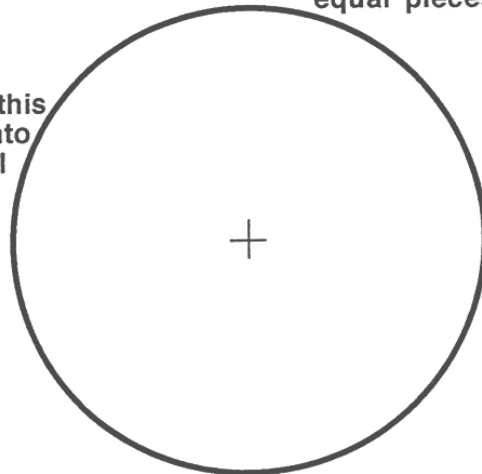


divide this
cake into 10
equal pieces



divide this
cake into 8
equal pieces

divide this
cake into
9 equal
pieces



Make a model cereal box

Problem Solving
draw a picture, make a model
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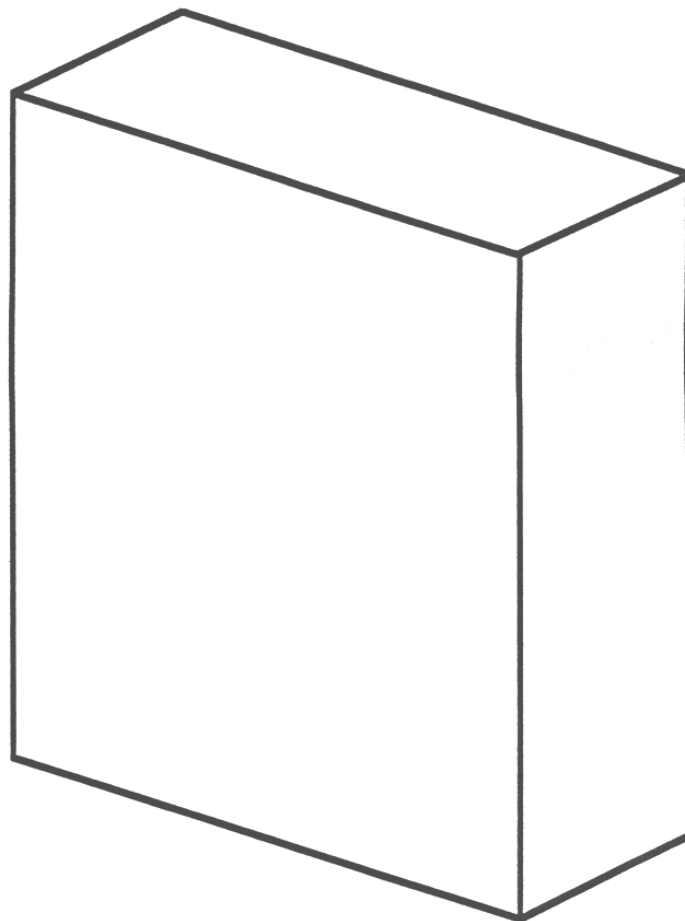
Get a cereal box, some card, scissors,
a ruler, and glue or sellotape.

Make a half size model of your cereal box.

Your cereal
box is a three
dimensional
object.

It has a
length,
breadth and a
height.

Your half size
model will
need to be
half size in
three
directions.



How many of your half size models
would fit into your original cereal box?
(there should be more than 2)

Handshakes

Every ——— is a handshake

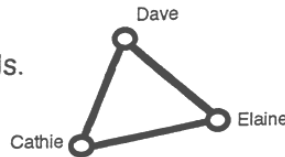
Andy and Brian meet and shake hands.

Only **1 handshake is needed for 2 people.**



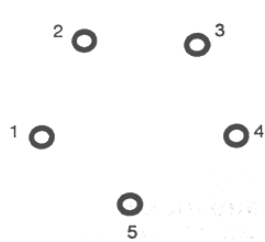
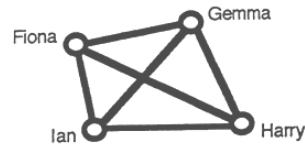
Cathie, Dave and Elaine meet and shake hands.

3 handshakes are needed for 3 people.

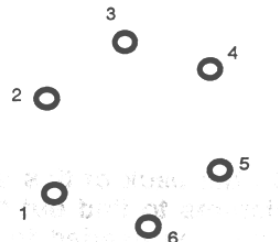


Fiona, Gemma, Harry and Ian meet and shake hands.

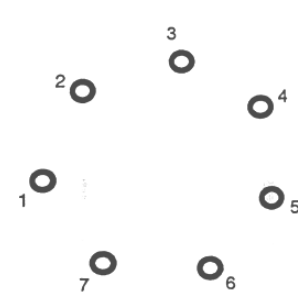
6 handshakes are needed for 4 people.



Use this diagram to see how many handshakes are needed for 5 people to be introduced.



Use this diagram to see how many handshakes are needed for 6 people to be introduced.



Use this diagram to see how many handshakes are needed for 7 people to be introduced.

Number of people	Number of handshakes
2	1
3	3
4	6
5
6
7
8
9
10

On the back of this sheet draw your own pictures to find out how many handshakes would be needed for 8, 9 and then 10 people.

Put all your results into the table.

Can you see a pattern?

Keep the pattern going to estimate how many handshakes would be needed for 20 people to be introduced.

Can you estimate how many handshakes would be needed for all the people in your class to be introduced to each other?

Making a gram

You need;

a newspaper
a weighing machine
a ruler
scissors

- * Find the weight your whole newspaper in grams
- * Find the weight of a sheet of the newspaper in grams

Use this data to cut out 1 gram of newspaper

Be as accurate as you can, and keep a record of your calculations to help justify why you think your sheet of paper weighs 1 gram.

Compare your gram of newspaper with other people's models.

Their models of 1 gram may be different shapes from yours, but if they are correct they should all have the same area.

This sheet of paper weighs about 5 grams.

Cut up this sheet to make a 1 gram model.

Now compare it to the 1 gram model you made from the newspaper.

The newspaper model of 1 gram will probably have a bigger area than the 1 gram model made from this paper.

Can you explain?

Keep these patterns going;

07:35, 07:50, 08:05,,,,,,

0.04m, 0.08m, 0.12m, 0.16m, 0.2m,,,,,

0.75kg, 0.8kg, 0.85kg,,,,,

12496, 12497, 12498,,,,,

24435, 24445, 24455,,,,,

300, 289, 278, 267,,,,,

£4.50, £4.62, £4.74,,,,,

15004, 15003, 15002,,,,,

15502, 15602, 15702,,,,,

1.4, 1.35, 1.3, 1.25,,,,,

£6.65, £7.70, £8.75, £9.80,,,,,

1, 2, 4, 8, 16, 32,,,,,

0.3, 1.1, 1.9, 2.7, 3.5,,,,,

14:33, 14:23, 14:13,,,,,

Keep these patterns going;

07:35, 08:00, 08:25, 08:50,,,,,

1.54m, 1.6m, 1.66m, 1.72m, 1.78m,,,,,

1.75kg, 1.6kg, 1.45kg,,,,,

32996, 32997, 32998,,,,,

4455, 4445, 4435,,,,,

4270, 4261, 4252,,,,,

£4.50, £4.59, £4.68,,,,,

5012, 5010, 5008,,,,,

15502, 15522, 15542,,,,,

1.35, 1.3, 1.25, 1.2,,,,,

£6, £6.14, £6.28, £6.42,,,,,

1, 2, 4, 7, 11, 16, 22,,,,,

1043, 1036, 1029,,,,,

14:33, 14:48, 15:03, 15:18,,,,,

Some patterns with multiplying;

$$\begin{aligned} 1 \times 2 &= \dots\dots\dots \\ 2 \times 3 &= \dots\dots\dots \\ 3 \times 4 &= \dots\dots\dots \\ 4 \times 5 &= \dots\dots\dots \\ 5 \times 6 &= \dots\dots\dots \\ 6 \times 7 &= \dots\dots\dots \\ 7 \times 8 &= \dots\dots\dots \\ 8 \times 9 &= \dots\dots\dots \\ 9 \times 10 &= \dots\dots\dots \end{aligned}$$

c
a
n

y
o
u

s
e
e

a

p
a
t
t
e
r
n
?

Describe the pattern you get in your own words:

.....
.....
.....
.....

Keep the pattern going;

$$\begin{aligned} 10 \times 11 &= \dots\dots\dots \\ 11 \times 12 &= \dots\dots\dots \\ 12 \times 13 &= \dots\dots\dots \\ 13 \times 14 &= \dots\dots\dots \\ 14 \times 15 &= \dots\dots\dots \\ 15 \times 16 &= \dots\dots\dots \\ 16 \times 17 &= \dots\dots\dots \\ 17 \times 18 &= \dots\dots\dots \\ 18 \times 19 &= \dots\dots\dots \\ 19 \times 20 &= \dots\dots\dots \end{aligned}$$

Check your answers with a calculator

Choose any odd number and any even number;

my odd number is

my even number is

Now let's multiply.

my odd number X my odd number makes, an number

my odd number X my even number makes, an number

my even number X my odd number makes, an number

my even number X my even number makes, an number

my even number X my even number makes, an number

my even number X my even number makes, an number

Complete this sentence;

The only time you get an odd answer when you multiply two whole numbers together is when

.....
.....

multiply sum odd or even?

43 x 49

57 x 72

68 x 71

84 x 96

Check your predictions with a calculator

Make the sums correct

Problem Solving
guess, check and improve
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Make these sums correct by using only 1.6, 2.4, 4, 8 or 10.

$$\square + \square = 6.4$$

$$\square \times \square = 24$$

$$\square - \square = 1.6$$

$$\square \div \square = 0.8$$

$$\square \times \square = 6.4$$

$$\square + \square = 11.6$$

$$\square \div \square = 0.2$$

$$\square - \square = 8.4$$

$$\square + \square = 4$$

$$\square \times \square = 12.8$$

$$\square - \square = 0.8$$

$$\square \div \square = 0.6$$

Make these sums correct by using only 0.5, 2.2, 3, 3.6, 5 or 10.

$$\square + \square = 6.6$$

$$\square \times \square = 22$$

$$\square - \square = 0.6$$

$$\square \div \square = 0.1$$

$$\square \times \square = 1.5$$

$$\square + \square = 10.5$$

$$\square \div \square = 0.5$$

$$\square - \square = 9.5$$

$$\square + \square = 2.7$$

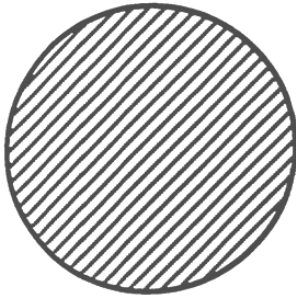
$$\square \times \square = 18$$

$$\square - \square = 2.8$$

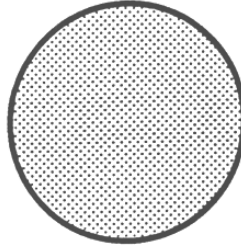
$$\square \div \square = 1.2$$

Make a kilogram

Problem Solving
guess, check and improve
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A striped ball
weighs
34 grams



A dotted ball
weighs
19 grams



A patterned ball
weighs
13 grams

Can you find a combination of types of balls which would weigh as near as possible to 1 kg?

eg
10 striped balls weigh
 $10 \times 34\text{g} = 340\text{g}$

12 dotted balls weigh
 $12 \times 19\text{g} = 228\text{g}$

15 patterned balls weigh
 $15 \times 13\text{g} = 195\text{g}$

Total weight = 763g,
which misses 1kg by 237g

or
15 striped balls weigh
 $15 \times 34\text{g} = 510\text{g}$

10 dotted balls weigh
 $10 \times 19\text{g} = 190\text{g}$

20 patterned balls weigh
 $20 \times 13\text{g} = 260\text{g}$

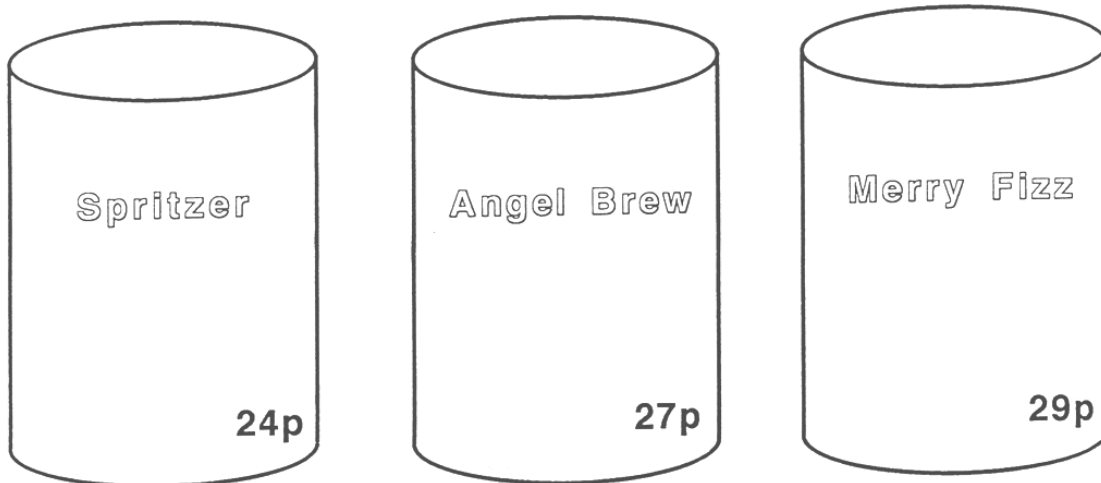
Total weight = 960g,
which only misses 1kg by 40g and is an improved combination.

Use the strategy 'guess, check and improve' and a calculator.

Least change from £10

Lucille is organising a party for her class.

She has £10 to spend and can buy 3 flavours of drinks for the party.



A can of Spritzer costs her 24p to buy, Angel Brew costs 27p, and Merry Fizz costs 29p.

She wants to spend as much of her £10 budget as possible.

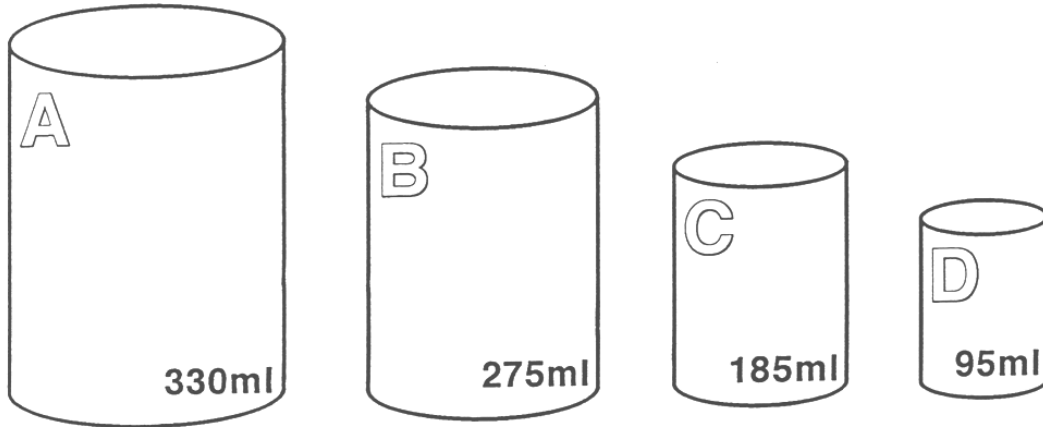
Can you find a combination of drinks Lucille could buy which would give the least possible change from £10?

eg 10 cans of Spritzer cost £2.40,
10 cans of Angel Brew cost £2.70
10 cans of Merry Fizz cost £2.90, and the total cost would come to £8, leaving £2 of unspent budget.

Use the strategy 'guess, check and improve', and a calculator.

Make 10 litres

Problem Solving
guess, check and improve
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Container A holds 330ml of liquid,
container B holds 275ml of liquid,
container C holds 185ml of liquid, and
container D holds 95ml of liquid.

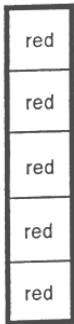
**Find a combination of
containers which would have
as near a volume as possible
of 10 litres.**

eg 10 of A would have $10 \times 330\text{ml} = 3300\text{ml} = 3.3 \text{ L}$
10 of B would have $10 \times 275\text{ml} = 2750\text{ml} = 2.75\text{L}$
10 of C would have $10 \times 185\text{ml} = 1850\text{ml} = 1.85\text{L}$
10 of D would have $10 \times 95\text{ml} = 950\text{ml} = 0.95\text{L}$

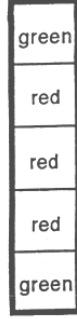
The total volume of this combination of containers would be 8.85L (which is 1.15L or 1150ml less than 10L).

**Use the strategy 'guess, check and improve',
and a calculator.**

Towers which are 5 high with 1 or 2 colours, and all different



Be systematic, what's missing?



Be systematic, what's missing?

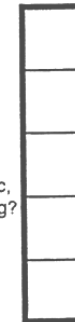


Be systematic, what's missing?



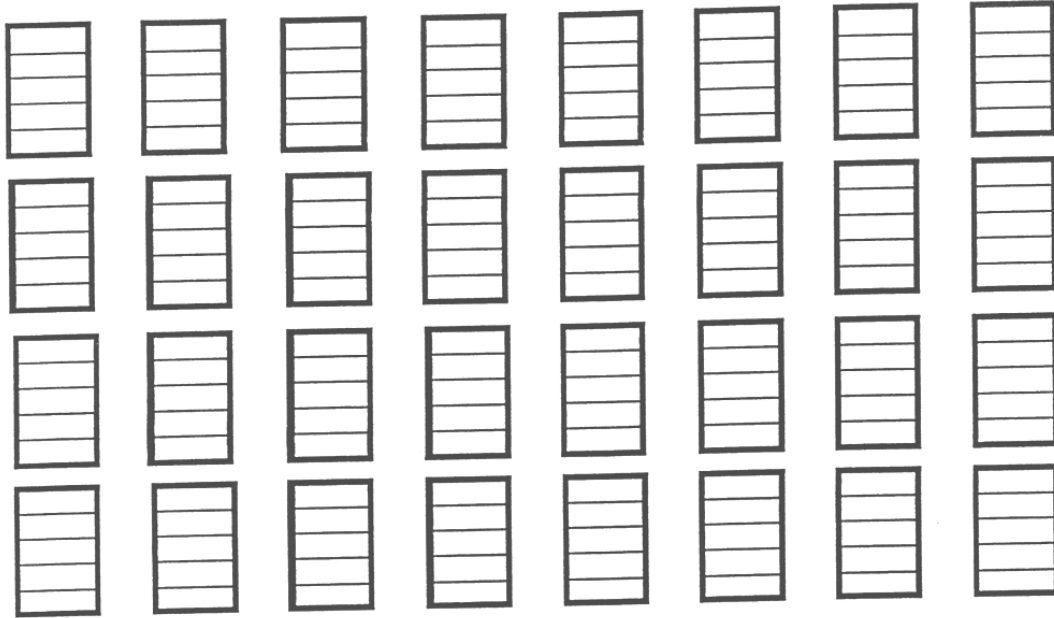
Be systematic, what's missing?

Be systematic, what's missing?

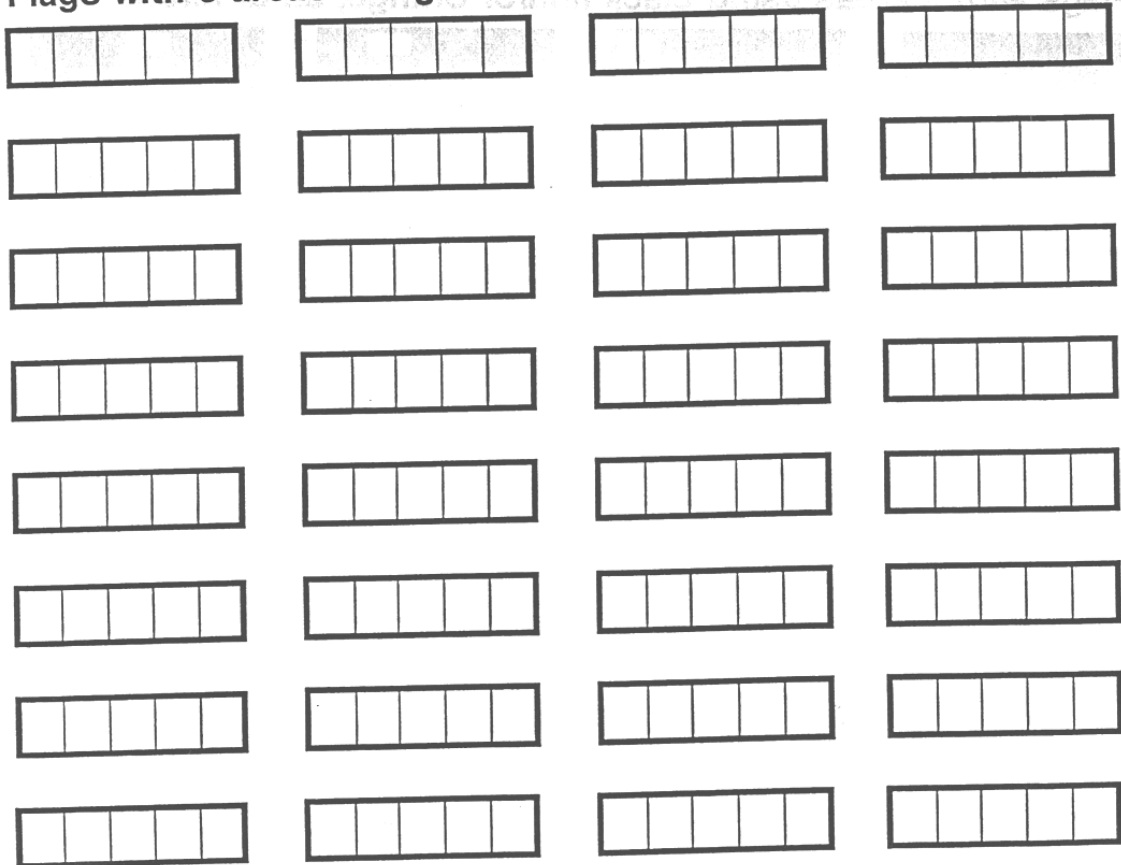


Towers which are 5 high using blue and/or yellow
and all different, be systematic to find as many as you can

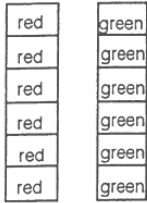
Problem Solving
be systematic
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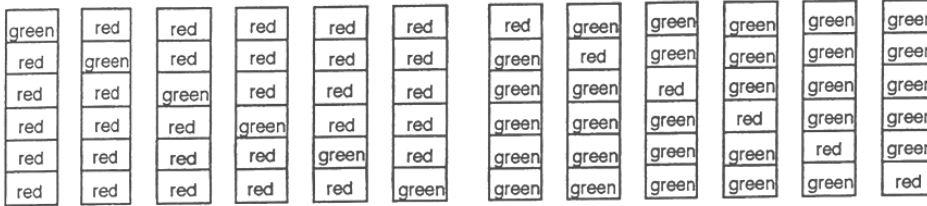
Flags with 5 areas using black and/or orange, all different



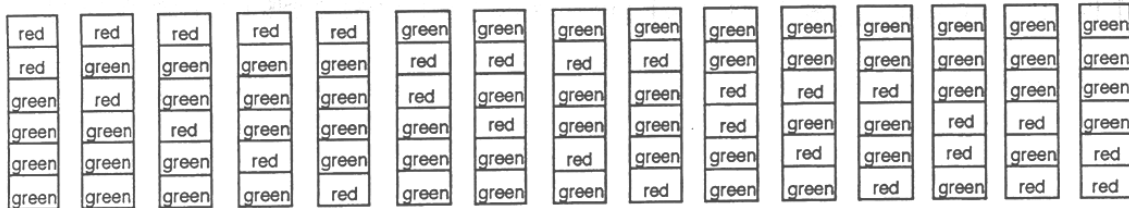
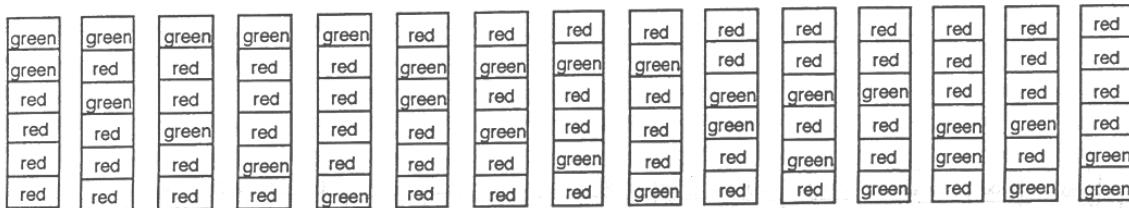
Towers which are 6 high using red and/or green, all different



6 of 1 colour
0 of 1 colour

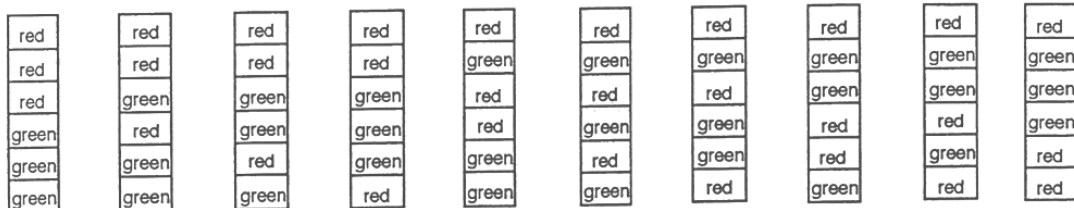
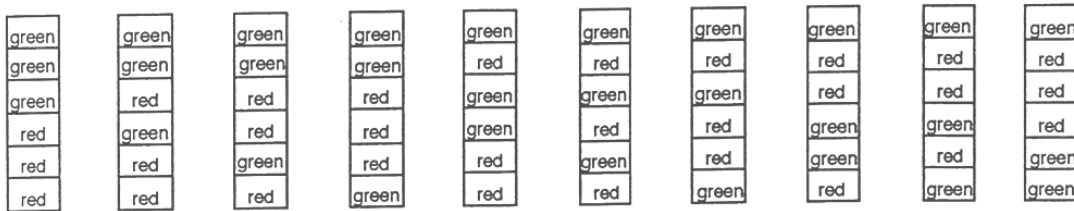


5 of 1 colour
1 of 1 colour



4 of 1 colour
2 of 1 colour

If you had 192 green cubes and 192 red cubes you could build 64 3D models of these towers.
All the 64 towers would be 6 cubes high, with red and/or green cubes, and each tower would be different.



3 of 1 colour
3 of 1 colour

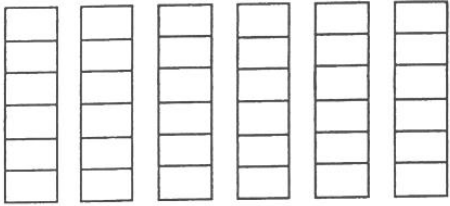


6 black and 0 orange

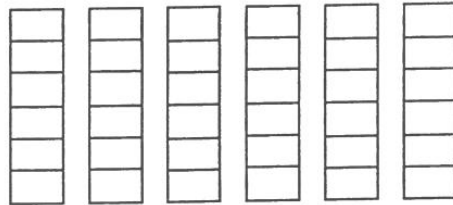


0 black and 6 orange

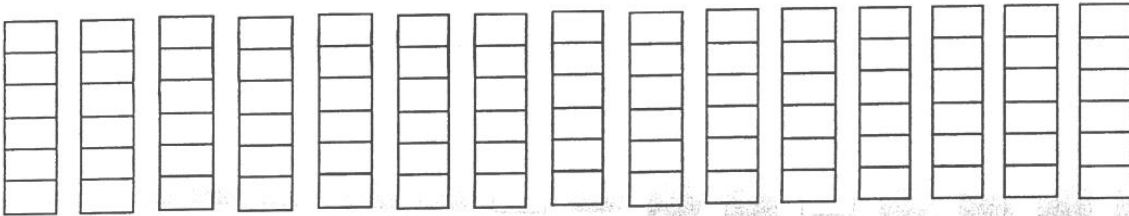
Towers which are 6 high using black and orange, all different



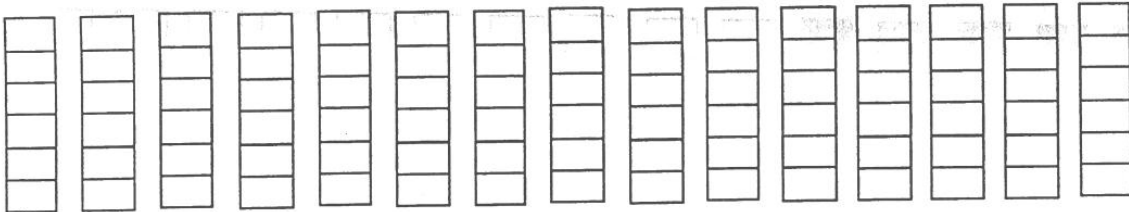
5 black and 1 orange



1 black and 5 orange

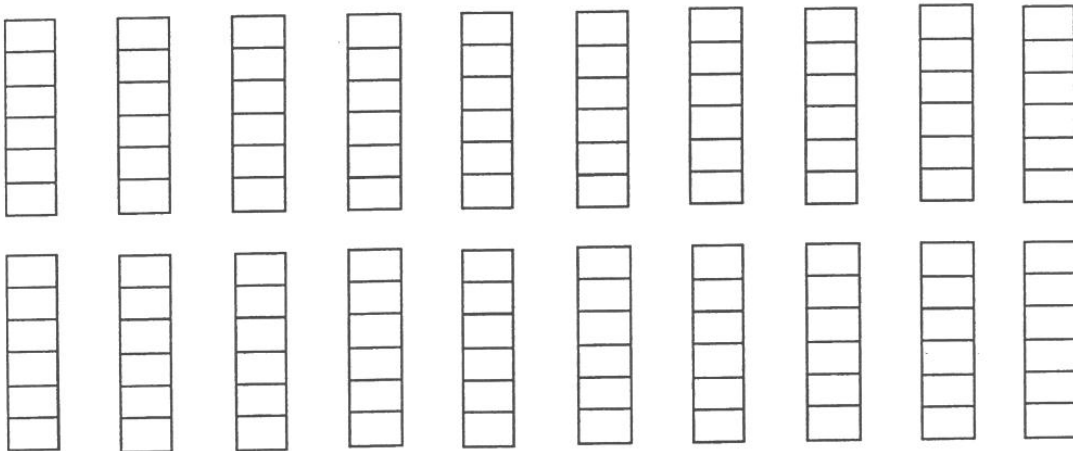


4 black and 2 orange



2 black and 4 orange

If you can be systematic it helps you find them all



3 black and 3 orange

Pizza tops

A shop sells take-away pizza.

It offers **2** types of topping, either **cheese** and/or **peppers**.

So you can order **3** types of pizza; * with cheese only
* with peppers only
* with both cheese and pepper

Another shop sells pizza but offers **3** types of topping, either **cheese** and/or **peppers** and/or **olives**. So you can order **7** types of pizza from this shop;

- * with cheese only
- * with peppers only
- * with olives only
- * with both cheese and peppers
- * with both cheese and olives
- * with both peppers and olives
- * with cheese, peppers and olives

Another shop wants to start selling pizza but offering more choice of toppings than the other shops.

They offer **4** types of topping;
cheese and/or **peppers** and/or **olives** and/or **salami**.

In your jotter, make a list of all the **15** possible pizzas that this shop can sell.

Be systematic, it will help you get them all.

Another shop-keeper sees that people like to eat pizza and have lots of choice, so they offer **5** types of topping;

cheese and/or
peppers and/or
olives and/or
salami and/or
anchovies.

In your jotter, make a list of all the **31** possible pizzas that this shop can sell.

Be systematic, it will help you get them all.

4, 8, 16, 32, 64, Can you see the pattern?

3, 7, 15, 31, 63, Can you see the pattern?

Crack the coded message

Take any book and open it at any page.
Find about any 100 words of text on the page.

Do a tally of the letters used in the 100 words in order to find out the most and least common letters used in the English language.

letter	tally
a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
p
q
r
s
t
u
v
w
x
y
z

Now use the results of your tally to 'crack the coded message';

9 3 2 £ > 7 7 ÷ ^ & § ! ^ &

.....

% > 9 3 2 % > 9 6 1 7 6 7

.....

£ > 9 9 2 & 5. 6 ! ≥ ^ 8

.....

1 > 5 7 2 2 £ > 9 9 2 & 5 7

.....

6 9 6 7 \$ 6 4 2 \$ ≥ 9 3 > 9

.....

≥ ^ 8 ÷ 6 \$ \$ + 2 @ ^ ^ §

.....

From your analysis;

* what is the most common letter used in the English language?

* what are the next few most common letters?

* what are some of the least used letters?

> 9 % > 9 3 7.

.....

Louise has 20p

Problem Solving
be systematic
(Level D) page 67

Louise has 20p, but we don't know how her 20p is made up.

It could be

$$10p + 5p + 2p + 2p + 1p$$

or

$$5p + 5p + 2p + 2p + 2p + 2p + 1p + 1p$$

or lots of other ways.

How many of the **41** ways can you find to make 20p?

You have to be systematic to find most or all of them.

24 different sequences with 4 different items

Put a    and  into each set of 4 boxes

in order to make each sequence different.

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Do
you
realise
that
 $1 \times 2 \times 3 \times 4$
 $= 24$?

What was the number?

Problem Solving
work backwards
(Level D) page 69

- A) I think of a number 'a', then double it.
My answer is 9. What is the number 'a'?
- B) I think of a number 'b', then multiply it by 4.
My answer is 48. What is the number 'b'?
- C) I think of a number 'c', then divide it by 3.
My answer is 7. What is the number 'c'?
- D) I think of a number 'd', then multiply it by 10.
My answer is 16. What is the number 'd'?
- E) I think of a number 'e', then divide it by 5.
My answer is 15. What is the number 'e'?
- F) I think of a number 'f', and then triple it.
My answer is 48. What is the number 'f'?
- G) I think of a number 'g', and then quarter it.
My answer is 16. What is the number 'g'?
- H) I think of a number 'h', then divide it by 10.
My answer is 3.2 so what is the number 'h'?
- I) I think of a number 'i', then divide it by 8.
My answer is 20. What is the number 'i'?
- J) I think of a number 'j', then halve it.
My answer is 6.5 so what is the number 'j'?
- K) I think of some money '£k', then double it.
My answer is £7.50. What is the amount of money '£k'?
- L) I think of a weight 'l grams', then divide it by 3.
My answer is 75 grams. What is the weight 'l grams'?
- M) I think of a volume 'm millilitres', then multiply it by 4.
My answer is 300 ml. What is the volume 'm millilitres'?

Make up your own problem for other people in your class to solve.

What was the number?

Problem Solving
work backwards
(Level D) page 70

- A) I think of a number 'a', double it, and then subtract 5.
My answer is 13. What is the number 'a'?
- B) I think of a number 'b', multiply it by 4, and then subtract 1.
My answer is 31. What is the number 'b'?
- C) I think of a number 'c', divide it by 3, and then subtract 4.
My answer is 2. What is the number 'c'?
- D) I think of a number 'd', multiply it by 10, and then add 3.
My answer is 53. What is the number 'd'?
- E) I think of a number 'e', divide it by 5, and then add 6.
My answer is 12. What is the number 'e'?
- F) I think of a number 'f', triple it, and then subtract 1.
My answer is 14. What is the number 'f'?
- G) I think of a number 'g', quarter it, and then add 2.
My answer is 6. What is the number 'g'?
- H) I think of a number 'h', multiply it by 6, and then subtract 10.
My answer is 32. What is the number 'h'?
- I) I think of a number 'i', divide it by 8, and then subtract 3.
My answer is 2. What is the number 'i'?
- J) I think of a number 'j', halve it, and then add 3.
My answer is 12. What is the number 'j'?
- K) I think of some money '£k', double it, and then subtract £1.50.
My answer is £6.50. What is the amount of money '£k'?
- L) I think of a weight 'l grams', divide it by 3, and then subtract 20 grams.
My answer is 0 grams. What is the weight 'l grams'?
- M) I think of a volume 'm millilitres', multiply it by 4, and then add 10 ml.
My answer is 70 ml. What is the volume 'm millilitres'?

'Working backwards'

(involving fractions)

Problem Solving
work backwards
(Level D) page 71

- A) Jenny had some money.
She spent £10, and then half of what she had left.
She now had £6 left altogether.
How much money did she have at the start?
- B) Bryce ate 400 grams of his cake at tea-time, then $\frac{1}{4}$ of what was left at supper-time. He now only had 300 grams of cake left.
How heavy was his cake to begin with?
- C) Syd had lots of sweets after her birthday party.
She ate 25, and sometime later she ate half of what was left.
She now had 30 sweets left.
How many sweets did she have to begin with?
- D) Louise had a bar of chocolate made up of small squares.
She ate half of the squares, then later she ate another 4 squares.
She now had 12 squares left.
How many squares did the chocolate bar have to begin with?
- E) In a primary 6 class 14 pupils go for school dinners, and two-thirds of the rest bring a pack lunch. The 6 remaining pupils all go home for lunch.
How many pupils are there in the P6 class altogether?
- F) There are four colours of marbles in a bag.
There are 100 red and 125 blue marbles.
Three-quarters of the rest of the marbles are yellow, and the remainder are green. If there are 10 green marbles in the bag how many marbles are there in the bag altogether?
- G) Sophie entered a sponsored run for charity. She ran 6km then got a stitch and walked for half the remaining distance. The stitch went away and she managed to run the last 4 km. How far was her sponsored run?
- H) Tessa spent £25 on a blouse, then three-quarters of what was left on a pair of shoes. She then had £5 left. How much did she have at the start?
- I) Ian spent two-thirds of his money on CD's and then another £4 on tapes. He then had £8 left. How much money did he have to start with?
- J) Two-thirds of a class were able to swim well, and two-thirds of the rest were able to swim a bit. Only 3 pupils were unable to swim at all. How many pupils are there in the class altogether?

**Clap your hands
a million times!**

**Estimate how long
it would it take for you
to clap your hands
1 million times.**

(take a simpler case than 1000000)

Extension;

**Estimate how long it
takes for you heart to
do 1 million beats.**

Letters and digits in a newspaper

You need a newspaper.

**Estimate how many
letters and digits
it takes to
write a newspaper.**

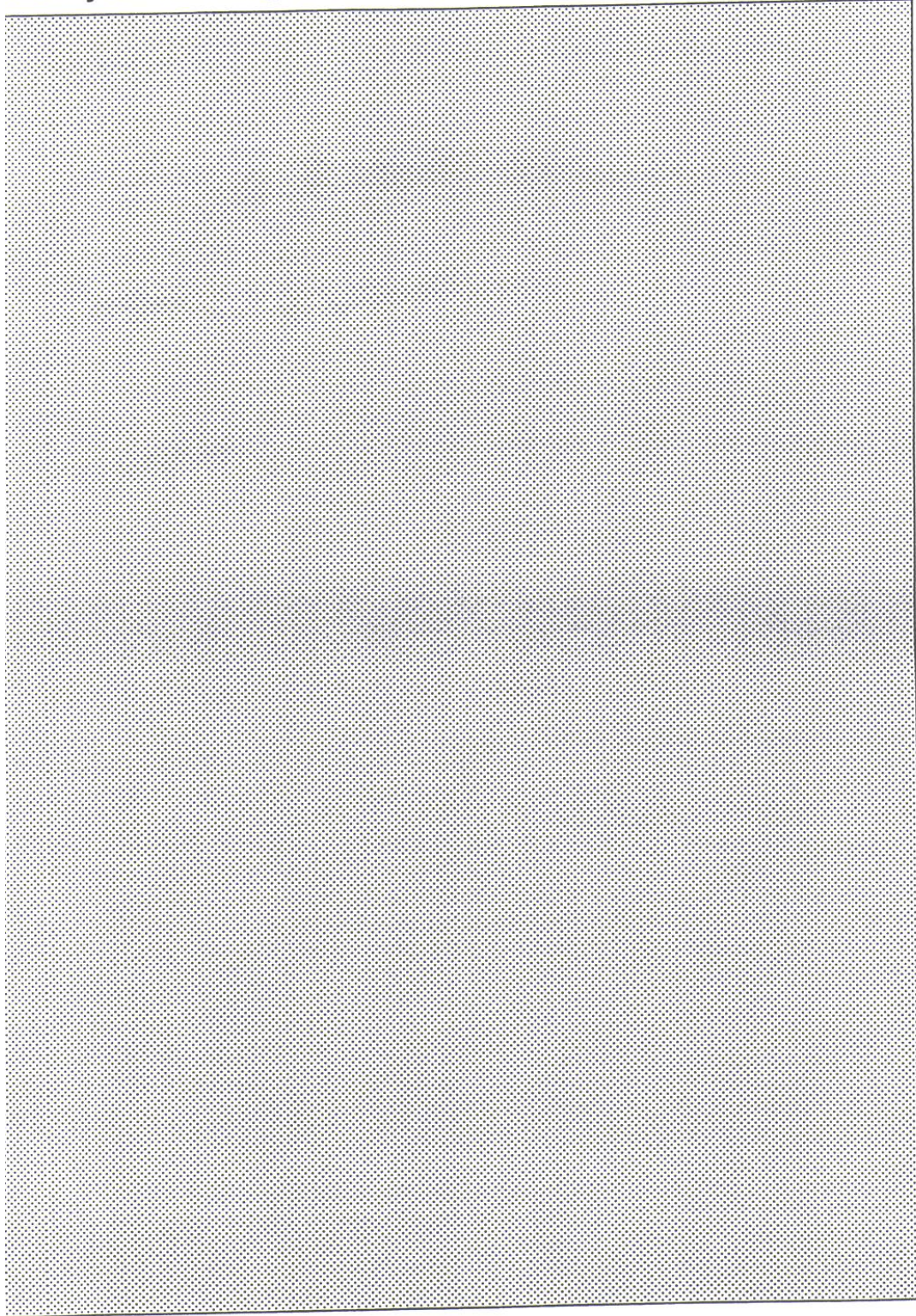
(take a simpler case
than the whole newspaper)

Extension;

**Find the book in the
school which has the
most words in it.**

show your working to justify your estimate)

(Level D) page 74

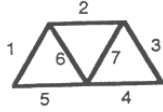


Straws and triangles

Problem Solving
take a simpler case,
make a table,
look for a pattern
(Level D) page 75

We can get lots of straws like this  and make triangles with them

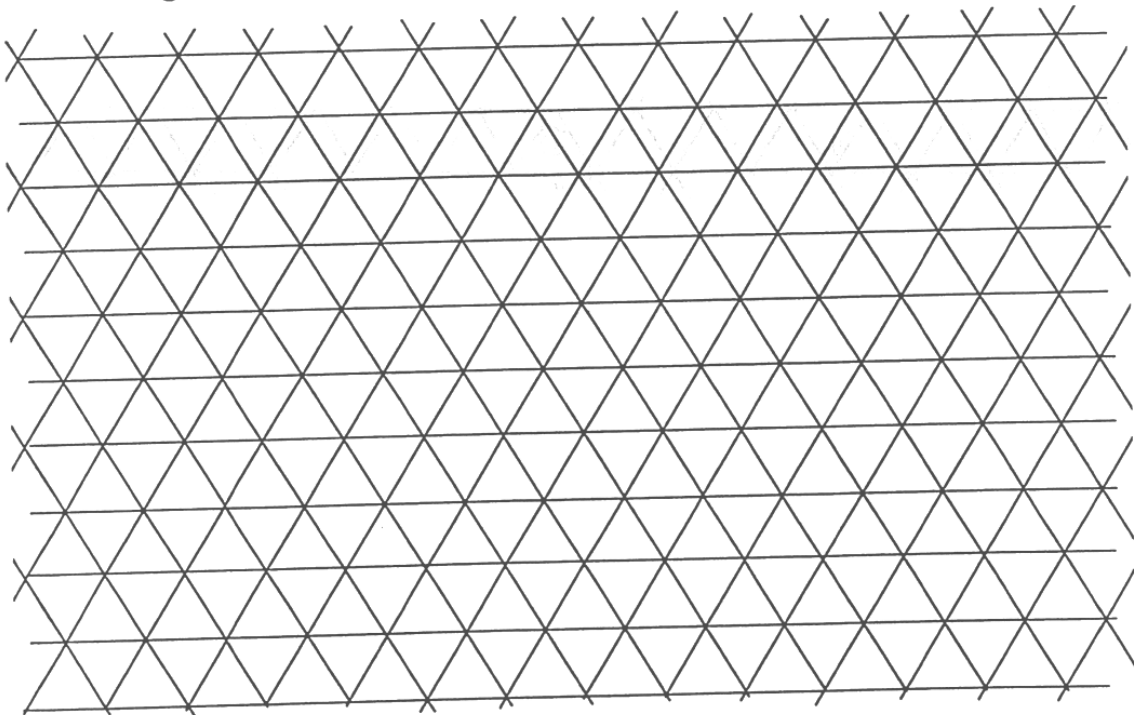
These 3
triangles need
7 straws



How many straws would be needed to make 100 triangles?

(Start with simpler cases, eg 1 triangle, then 2 triangles, then 3 and more, make a table of your results and see if you can spot a pattern that you can keep going for 100 triangles)

Use the grid below to help you draw the simpler cases.



- Can you predict how many straws 100 triangles would need?
- Can you predict how many straws 1000 triangles would need?
- Can you predict how many straws 10000 triangles would need?
- Can you predict how many straws 100000 triangles would need?
- Can you predict how many straws 1 million triangles would need?

Straws and squares

Problem Solving
take a simpler case,
make a table,
look for a pattern
(Level D) page 76

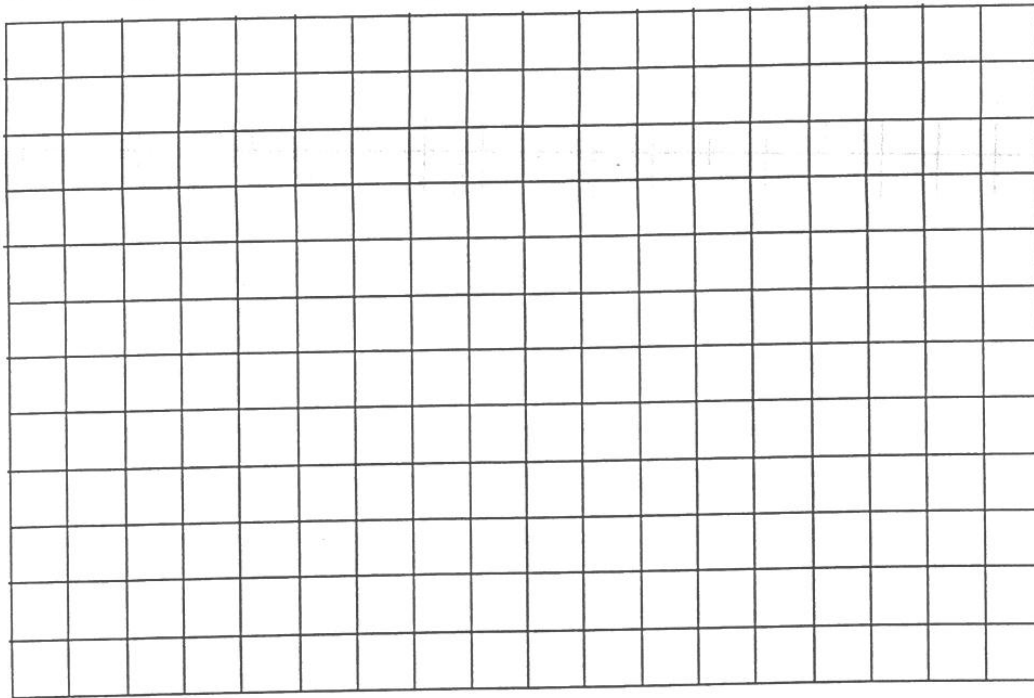
We can get lots of straws like this  and make squares with them



How many straws would be needed to make 100 squares?

(Start with simpler cases, eg 1 square, then 2 squares, then 3 and more, make a table of your results and see if you can spot a pattern that you can keep going for 100 squares)

Use the grid below to help you draw the simpler cases.

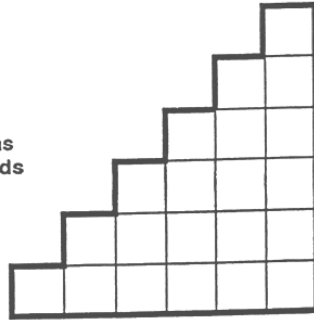


- Can you predict how many straws 100 squares would need?
- Can you predict how many straws 1000 squares would need?
- Can you predict how many straws 10000 squares would need?
- Can you predict how many straws 100000 squares would need?
- Can you predict how many straws 1 million squares would need?

Stairways

Problem Solving
take a simpler case,
make a table,
look for a pattern
(Level D) page 77

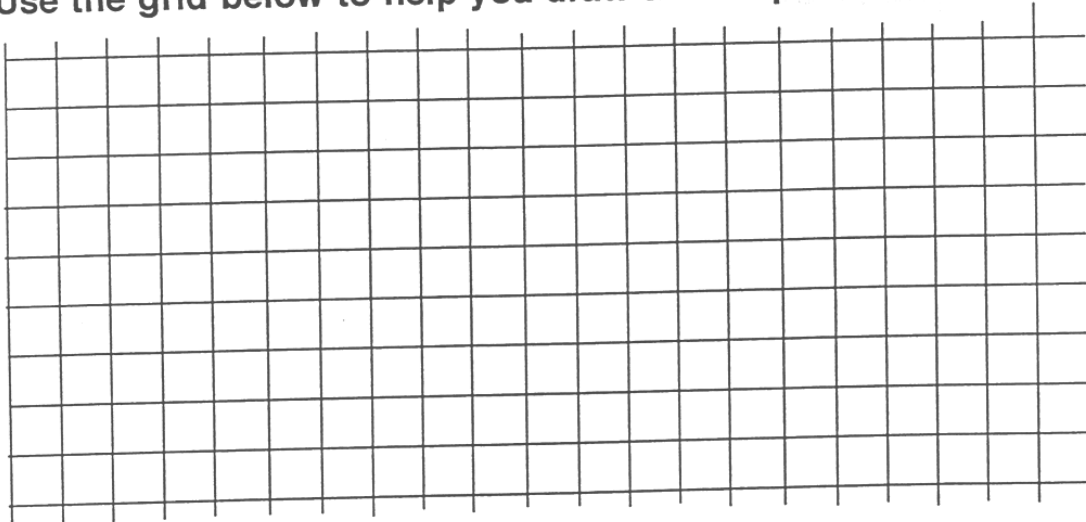
This stairway has
6 steps and needs
21 cubes



How many cubes would be needed to make a 30 high stairway?

Start with a simpler case, eg a stair 1 high then a stair 2 high, then 3 or more, make a table of your results and see if you can spot a pattern that you can keep going for a 30 high stairway.

Use the grid below to help you draw the simpler cases.



- Can you predict how many cubes a 30 high stair would need?
- Can you predict how many cubes a 40 high stair would need?
- Can you predict how many cubes a 50 high stair would need?
- Can you predict how many cubes a 100 high stair would need?